

Exponents and Place Value

base \longrightarrow 5^4 \longleftarrow exponent

The number 5 is the **base**. The base is the factor that is being multiplied.

The number 4 is the **exponent**. The exponent tells how many times the base is used as a factor.

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

The base (5) is used as a factor the exponent (4) number of times.

To write a product in exponential form:

$$4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$$

First write the base: **4**

Count the number of times the base is used as a factor. This is the exponent. **4^7**

To evaluate an exponential number: **6^3**

Write the base as a factor the number of times shown by the exponent.

$$6^3 = 6 \times 6 \times 6 = 216$$

To write the expanded form of a number using exponents:

Write the number in expanded form.

$$52,965 = (5 \times 10,000) + (2 \times 1,000) + (9 \times 100) + (6 \times 10) + (5 \times 1)$$

Write the place values as powers of 10.

$$52,965 = (5 \times 10^4) + (2 \times 10^3) + (9 \times 10^2) + (6 \times 10^1) + (5 \times 10^0)$$

Tip: Any number raised to the first power equals that number. **$8^1 = 8$**

Write each power as a product and evaluate the expression.

1. 9^4 _____ 2. 4^5 _____

Write each product in exponential form.

3. $3 \times 3 \times 3 \times 3 \times 3$ _____ 4. $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ _____

Write the number in expanded form using exponents.

5. $74,271 =$ _____ $+$ _____ $+$ _____ $+$ _____ $+$ _____

6. **Number Sense** Explain the difference between 4^6 and 6^4 .

Using Variables to Write Expressions

A variable represents a quantity that can change. To use a variable to write an algebraic expression for a situation, you need to decide which operation is appropriate for the situation. To help you, some words and phrases are listed below.

Word phrase	Variable	Operation	Algebraic Expression
ten more than a number b	b	Addition	$b + 10$
the sum of 8 and a number c	c		$8 + c$
five less than a number d	d	Subtraction	$d - 5$
15 decreased by a number e	e		$15 - e$
the product of 8 and a number f	f	Multiplication	$8f$
19 times a number g	g		$19g$
the quotient of a number h divided by 2	h	Division	$h \div 2$
a number i divided into 50	i		$50 \div i$

Write each algebraic expression.

1. a number j **divided by** 5

Identify the operation.

Write the expression.

2. the **sum** of 2 and a number k

3. 6 **times** a number m

4. a number n **divided into** 9

5. 4 **less than** a number p

6. q fewer times than 10

7. r tickets at \$7 each

8. A field goal scores 3 points. Write an algebraic expression to represent the number of points the Raiders will score from field goals.

Identify the operation

Write the expression.

9. **Writing to Explain** Write an algebraic expression to represent the situation below. Explain how the expression relates to the situation.

Some children share 5 apples equally among themselves.

Order of Operations

Order of operations is a set of rules that mathematicians use when computing numbers. Here is how order of operations is used to solve the following problem: $7 + (5 \times 4) \times 3$.

Order of Operations

First, compute all numbers inside parentheses.

$$7 + (5 \times 4) \times 3$$

$$7 + 20 \times 3$$

Next, evaluate terms with exponents. If there are no exponents, go to the next step.

$$7 + 20 \times 3$$

Then, multiply and divide the numbers from left to right.

$$7 + 60$$

Finally, add and subtract the numbers from left to right.

$$67$$

How to use parentheses to make each sentence true:

$$6 + 2 \times 9 = 72$$

Using order of operations,
 $6 + 2 \times 9 = 24$, not 72.

Place parentheses around $6 + 2$ so that this operation is done first:

$$(6 + 2) \times 9 = 72$$

$$8 \times 9 = 72$$

Evaluate each expression.

1. $8 + 7 \times 5 =$ _____

2. $18 - 3 \times 2 =$ _____

3. $3 \times 7 + 3 \times 5 =$ _____

4. $40 \div (2 \times 4) =$ _____

5. $6 \times 3 - 6 \times 2 =$ _____

6. $9 + 2^3 =$ _____

7. $7 + 12 \times 3 - 2 =$ _____

8. $4 \times (5 + 5) \div 20 + 6 =$ _____

9. $4^2 - (3 \times 5) =$ _____

10. $(3 \times 2) + 3^2 =$ _____

11. **Reasoning** Which operation should be performed *last* in this problem: $3^2 + 7 \times 4$? Why?

Use parentheses to make each sentence true.

12. $0 \times 6 + 9 = 9$ _____

13. $3^2 + 2 \times 2 = 13$ _____

Evaluating Expressions

To evaluate an expression, follow these steps:

1. Substitute or replace the variable with the value given in the problem.
2. Perform the operation or operations.
3. If there is more than one operation, use the order of operations.

Evaluate $4 + 2n$ for 3.

Replace n with 3.

$$4 + 2(3)$$

Multiply first.

$$4 + 6$$

Then add.

$$10$$

The value of the expression is 10.

Evaluate $g^2 - 3(3) + g \div 2$; $g = 4$.

Replace g with 4.

$$4^2 - 3(3) + 4 \div 2$$

Evaluate terms with exponents.

$$16 - 3(3) + 4 \div 2$$

Then multiply and divide.

$$16 - 9 + 2$$

Then subtract and add.

$$9$$

The value of the expression is 9.

Apply the substitutions and evaluate.

1. $12n$; $n = 3$

2. $2t - 4$; $t = 6$

3. $r + 48 \div r$; $r = 8$

For **4–7**, evaluate each expression for 3, 6, and 8.

4. $7x$ _____, _____, _____

5. $6x + 4$ _____, _____, _____

6. $14 + x \div 2$ _____, _____, _____

7. $x + 2x$ _____, _____, _____

8. Katie rented a bicycle at the beach for \$3 an hour plus a \$5 fee.

Write an expression that shows how much it will cost Katie to rent the bicycle. Then solve the expression for 4 hours.

9. **Writing to Explain** Timothy is solving the problem $50 + 108x \div 4$.

What order of operations should he follow?

Multiplying Decimals

Use the same strategy to multiply a decimal by a whole number or to multiply a decimal by a decimal.

Multiply 0.72×23 .

Ignore the decimal points. Multiply as you would with two whole numbers.

Count the number of decimal places in both factors. Use that number of decimal places to write the answer.

$$\begin{array}{r} 0.72 \\ \times 23 \\ \hline 216 \\ 144 \\ \hline 1656 \\ 16.56 \end{array}$$

2 decimal places

Multiply 0.45×0.8 .

Ignore the decimal points. Multiply as you would with two whole numbers.

Count the number of decimal places in both factors. Use that number of decimal places to write the answer.

$$\begin{array}{r} 0.45 \\ \times 0.8 \\ \hline 360 \\ 0.360 \end{array}$$

$2 + 1 = 3$ decimal places

Place the decimal point in each product.

1. $1.2 \times 3.6 = 432$

2. $5.5 \times 3.77 = 20735$

3. $4.4 \times 2.333 = 102652$

Find the product.

4. 7×0.5 _____

5. 12×0.08 _____

6. 24×0.17 _____

7. 0.4×0.17 _____

8. 1.9×0.46 _____

9. 3.42×5.15 _____

10. **Writing to Explain** If you multiply two decimals less than 1, can you predict whether the product will be less than or greater than either of the factors? Explain.

11. **Number Sense** Two factors are multiplied and their product is 34.44. One factor is a whole number. How many decimal places are in the other factor?

Dividing Decimals

When you divide by a decimal, you need to rewrite the dividend and the divisor so that you are dividing by a whole number.

Find $2.48 \div 0.8$.

$$240 \div 80 = 3$$

Step 1: Estimate. Use compatible numbers.

Step 2: Make the divisor a whole number. Multiply the divisor AND the dividend by the same power of 10.

Place the decimal in the quotient.

Step 3: Divide as you would with whole numbers. Remember that sometimes you may need to annex zeros to complete your division.

Step 4: Compare the quotient with your estimate.

The diagram illustrates the process of dividing a decimal by another decimal. It starts with the problem $0.8 \overline{)2.48}$. A cloud-shaped callout shows the conversion: $0.8 \times 10 = 8$ and $2.48 \times 10 = 24.8$. An arrow points to the equivalent whole-number division $8 \overline{)24.8}$. The long division is shown as follows:

$$\begin{array}{r} 3.1 \\ 8 \overline{)24.8} \\ \underline{24} \\ 8 \\ \underline{8} \\ 0 \end{array}$$

Since 3.1 is close to 3, the answer checks.

Find each quotient.

1. $0.2 \overline{)1.5}$

Estimate: _____

Multiply dividend and divisor by what power of 10? _____

Place the decimal point in the quotient.

Divide. How many zeros do you need to annex? _____

Compare the quotient to your estimate.

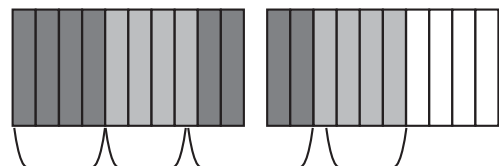
Is the answer reasonable? _____

2. $0.6 \overline{)0.36}$

3. $0.4 \overline{)9.6}$

4. $0.75 \overline{)0.3}$

5. **Draw a Picture** Fernando used tenths grids to draw this picture showing $1.6 \div 0.4 = 4$. Draw a picture to show $1.8 \div 0.6$. Write the quotient.



Fractions and Decimals

A fraction and a decimal can both be used to represent the same value.

Write 0.35 as a fraction.

Write the decimal as a fraction with a denominator of 10, 100, 1000, or another power of ten.

$$0.35 = 35 \text{ hundredths} = \frac{35}{100}$$

Then write the fraction in simplest form.

$$\frac{35}{100} = \frac{35 \div 5}{100 \div 5} = \frac{7}{20}$$

$$\text{So } 0.35 = \frac{7}{20}.$$

Write $\frac{3}{25}$ as a decimal.

Method 1: Write an equivalent fraction with a denominator of 10, 100, 1000, or another power of ten. Then write the decimal.

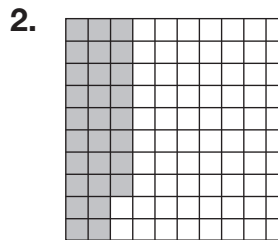
$$\frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = 0.12$$

Method 2: Divide the numerator by the denominator.

$$\text{So } \frac{3}{25} = 0.12.$$

$$\begin{array}{r} 0.12 \\ 25 \overline{)3.00} \\ \underline{-25} \\ 50 \\ \underline{-50} \\ 0 \end{array}$$

Write a decimal and a fraction in simplest form for each shaded portion.



Write each decimal as a fraction in simplest form.

3. 0.5 _____ 4. 0.8 _____ 5. 0.36 _____

6. 0.25 _____ 7. 0.125 _____ 8. 0.070 _____

Convert each fraction to a decimal.

9. $\frac{93}{100}$ _____ 10. $\frac{7}{10}$ _____ 11. $\frac{11}{20}$ _____

12. $\frac{14}{25}$ _____ 13. $\frac{7}{40}$ _____ 14. $\frac{6}{100}$ _____

15. **Geometry** Draw eight congruent figures. Shade some of the figures to make a color pattern. Write a decimal and a fraction in simplest form to represent the shaded part of the set.

Improper Fractions and Mixed Numbers

A mixed number combines a whole number with a fraction. It is greater than one.

An improper fraction has a numerator that is larger than its denominator.

How to Write an Improper Fraction as a Mixed Number

Write $\frac{12}{5}$ as a mixed number.

Divide the numerator by the denominator.

The quotient is the whole number in the mixed number.

$$\begin{array}{r} 2 \\ 5 \overline{)12} \\ \underline{-10} \\ 2 \end{array}$$

The remainder is the numerator.
The denominator stays the same.

$$\frac{12}{5} = 2\frac{2}{5}$$

How to Write a Mixed Number as an Improper Fraction

Multiply the denominator by the whole number.

$$3\frac{2}{5}$$

$$5 \times 3 = 15$$

Then add the numerator. $15 + 2 = 17$

Write this number for the numerator. $\rightarrow \frac{17}{5}$
Use the original denominator. $\rightarrow \frac{17}{5}$

$$3\frac{2}{5} = \frac{17}{5}$$

1. Draw a picture to show $4\frac{2}{3}$.

Write each improper fraction as a whole number or mixed number in simplest form.

2. $\frac{60}{40}$ _____

3. $\frac{33}{10}$ _____

4. $\frac{12}{7}$ _____

Write each mixed number as an improper fraction.

5. $4\frac{1}{3}$ _____

6. $1\frac{20}{50}$ _____

7. $8\frac{7}{8}$ _____

8. **Reasoning** Write 6 as an improper fraction with a denominator of 10. _____

Adding and Subtracting: Unlike Denominators

If you are adding or subtracting fractions and the denominators are not the same, the first thing to do is find a common denominator. The best common denominator to use is the least common multiple of the two denominators.

Step 1:

Use the LCM to find a common denominator.

$$\text{Find } \frac{2}{6} + \frac{1}{2}.$$

The LCM of 2 and 6 is 6.
The least common denominator (LCD) is 6.

$$\text{Find } \frac{3}{4} - \frac{1}{3}.$$

The LCD of 3 and 4 is 12.

Step 2:

Write equivalent fractions.

$$\begin{array}{r} \frac{2}{6} = \frac{2}{6} \\ + \frac{1}{2} = + \frac{3}{6} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ - \frac{1}{3} = - \frac{4}{12} \\ \hline \end{array}$$

Step 3:

Add or subtract.
Simplify if possible.

$$\begin{array}{r} \frac{2}{6} = \frac{2}{6} \\ + \frac{1}{2} = + \frac{3}{6} \\ \hline \frac{5}{6} \end{array}$$

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ - \frac{1}{3} = - \frac{4}{12} \\ \hline \frac{5}{12} \end{array}$$

Find each sum or difference. Simplify your answer.

1. $\frac{3}{4} + \frac{5}{2} =$ _____

2. $\frac{11}{12} - \frac{1}{3} =$ _____

3. $\frac{4}{15} + \frac{4}{5} =$ _____

4. $\frac{5}{6} - \frac{4}{9} =$ _____

5. $\frac{2}{3} + \frac{7}{10} =$ _____

6. $\frac{2}{5} + \frac{2}{3} - \frac{6}{30} =$ _____

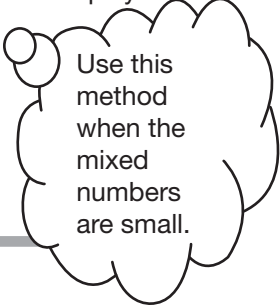
7. **Number Sense** The least common denominator for the sum $\frac{3}{8} + \frac{5}{12}$ is 24. Name another common denominator that you could use.
- _____

8. A recipe calls for $\frac{1}{2}$ cup of milk and $\frac{1}{3}$ cup of water. What is the total amount of liquid in the recipe?
- _____

Subtracting Mixed Numbers

To subtract mixed numbers, the fractional parts must have the same denominator. Use one of these methods:

Step 1	Step 2	Step 3	Step 4
Find $8\frac{1}{3} - 5\frac{4}{5}$ Estimate: $8 - 6 = 2$	Use the LCD to write equivalent fractions. $8\frac{1}{3} = 8\frac{5}{15}$ $5\frac{4}{5} = 5\frac{12}{15}$	Rename $8\frac{5}{15}$ to show more fifteenths so you can subtract. $8\frac{5}{15}$ $7\frac{5}{15} + \frac{15}{15}$ $7\frac{20}{15}$	Subtract and simplify if possible. $7\frac{20}{15} - 5\frac{12}{15} = 2\frac{8}{15}$
Find $3\frac{1}{2} - 1\frac{5}{8}$ Estimate: $4 - 2 = 2$	Write each mixed number as an improper fraction. $3\frac{1}{2} = \frac{7}{2}$ $1\frac{5}{8} = \frac{13}{8}$	Use the LCD to rewrite the improper fractions with the same denominator. $\frac{7}{2} = \frac{28}{8}$ $\frac{13}{8}$	Subtract and simplify if possible. $\frac{28}{8} - \frac{13}{8} = \frac{15}{8} = 1\frac{7}{8}$



Use this method when the mixed numbers are small.

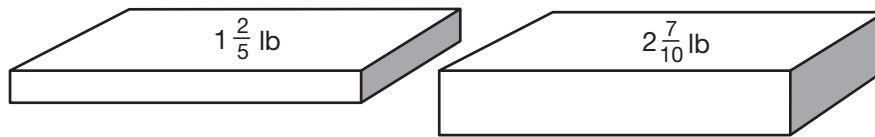
Find each difference. Simplify if possible.

1. $5\frac{9}{10} - 2\frac{3}{5} =$ _____
2. $11\frac{7}{16} - 8\frac{3}{8} =$ _____
3. $9\frac{2}{3} - 9\frac{1}{6} =$ _____
4. $4\frac{2}{3} - 2 =$ _____
5. $4\frac{1}{4} - \frac{7}{12} =$ _____
6. $5\frac{6}{7} - 2\frac{13}{14} =$ _____
7. $6\frac{5}{16} - 3\frac{3}{4} =$ _____
8. $8 - 4\frac{7}{10} =$ _____
9. $2\frac{1}{5} - \frac{13}{15} =$ _____
10. $7\frac{7}{8} - 2\frac{3}{4} =$ _____
11. $3\frac{1}{3} - 1\frac{7}{9} =$ _____
12. $12\frac{3}{8} - 5\frac{1}{8} =$ _____
13. $7\frac{3}{4} - 2\frac{7}{8} =$ _____
14. $3\frac{7}{9} - 1\frac{1}{3} =$ _____
15. $12\frac{1}{8} - 5\frac{3}{8} =$ _____

16. **Number Sense** How do you know if you need to rename the first number in a subtraction problem involving mixed numbers?

Adding Mixed Numbers

You can add to find the total weight of these two packages of cheese.



Write the fractions so they both have the same denominator. Add the whole numbers. Add the fractions.

$$\begin{array}{r}
 1\frac{2}{5} = 1\frac{4}{10} \\
 + 2\frac{7}{10} = + 2\frac{7}{10} \\
 \hline
 3\frac{11}{10}
 \end{array}
 = 3 + 1\frac{1}{10} = 4\frac{1}{10}$$

Write the improper fraction as a mixed number. Add the whole numbers. Write the fraction in simplest form.

The total weight of the cheese is $4\frac{1}{10}$ pounds.

Find each sum. Simplify your answer.

$$\begin{array}{r}
 1. \quad 5\frac{2}{3} = 5\frac{4}{6} \\
 + 3\frac{1}{6} = + 3\frac{1}{6} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 2. \quad 7\frac{4}{5} = 7\frac{8}{10} \\
 + 6\frac{1}{4} = + 6\frac{2.5}{10} \\
 \hline
 \end{array}$$

$$3. \quad 8\frac{7}{11} + 14\frac{6}{11} = \underline{\hspace{2cm}}$$

$$4. \quad 6\frac{1}{4} + 9\frac{7}{8} = \underline{\hspace{2cm}}$$

$$5. \quad 3\frac{5}{8} + 12\frac{1}{6} = \underline{\hspace{2cm}}$$

$$6. \quad 14 + 13\frac{5}{7} = \underline{\hspace{2cm}}$$

7. On Monday, $3\frac{7}{10}$ inches of snow fell during the day. Another $5\frac{1}{2}$ inches of snow fell that night. What was the total snowfall?

8. **Writing to Explain** Explain how to rewrite $5\frac{7}{8} + 14\frac{1}{6}$ so the fractions have the same denominator. Find the sum.

Multiplying Fractions

Find $\frac{3}{4} \times \frac{2}{5}$.

Draw a picture.

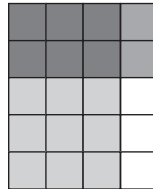
Shade the squares.

There are 20 squares in all.

6 squares have overlapping shading.

$$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$$

$$\text{Simplify: } \frac{6}{20} = \frac{3}{10}$$



Use the denominators to determine the number of squares: 5 tall and 4 wide.

Multiply the numerators and the denominators. Simplify if possible.

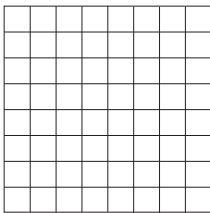
$$\begin{aligned} \frac{3}{4} \times \frac{2}{5} &= \frac{(3 \times 2)}{(4 \times 5)} \\ &= \frac{6}{20} \\ &= \frac{3}{10} \end{aligned}$$

Simplify first. Divide a numerator and a denominator by their GCF. Then multiply.

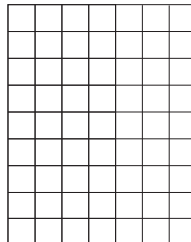
$$\begin{aligned} \frac{3}{4} \times \frac{2}{5} &= \frac{3}{(4 \div 2)} \times \frac{(2 \div 2)}{5} \\ &= \frac{3}{2} \times \frac{1}{5} \\ &= \frac{3}{10} \end{aligned}$$

Draw a picture to solve.

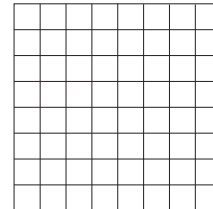
1. $\frac{1}{3} \times \frac{4}{5} =$ _____



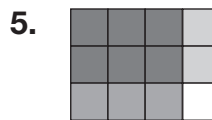
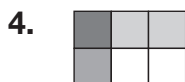
2. $\frac{3}{4} \times \frac{3}{9} =$ _____



3. $\frac{1}{2} \times \frac{7}{8} =$ _____



Write an equation for each picture.



Find each product. Simplify if possible.

6. $\frac{5}{7} \times \frac{3}{10} =$ _____

7. $\frac{1}{2} \times \frac{6}{15} =$ _____

8. $\frac{4}{7} \times \frac{1}{2} =$ _____

9. $\frac{5}{6} \times \frac{3}{8} =$ _____

10. $\frac{6}{7} \times \frac{5}{12} =$ _____

11. $8 \times \frac{3}{4} =$ _____

12. **Number Sense** Can you simplify before multiplying $14 \times \frac{4}{7}$? Explain.

Multiplying Mixed Numbers

How to find the product of two mixed numbers: Find $3\frac{2}{3} \times 4\frac{1}{2}$.

Step 1

Estimate the product by rounding.

Round $3\frac{2}{3}$ to 4 and $4\frac{1}{2}$ to 5:
 $4 \times 5 = 20$

Step 2

Write each mixed number as an improper fraction.

$$3\frac{2}{3} = \frac{11}{3} \text{ and } 4\frac{1}{2} = \frac{9}{2}$$

Look for common factors and simplify.

$$3\frac{2}{3} \times 4\frac{1}{2} = \frac{11}{\cancel{3}_1} \times \frac{\cancel{9}^3}{2} = \frac{11}{1} \times \frac{3}{2}$$

Step 3

Multiply the numerators and denominators.

$$\frac{11}{1} \times \frac{3}{2} = \frac{33}{2}$$

Write the product as a mixed number.

$$\frac{33}{2} = 16\frac{1}{2}$$

$16\frac{1}{2}$ is close to 20, so the answer is reasonable.

Find each product. Simplify if possible.

1. $2\frac{3}{4} \times 3\frac{1}{2}$ _____ 2. $2\frac{1}{5} \times 2\frac{2}{3}$ _____ 3. $6 \times 3\frac{1}{4}$ _____

4. $1\frac{2}{5} \times 3\frac{1}{4}$ _____ 5. $4\frac{1}{2} \times 16$ _____ 6. $1\frac{3}{8} \times 2\frac{1}{2}$ _____

Evaluate each expression for $K = 2\frac{1}{3}$.

7. $12K$ _____ 8. $1\frac{3}{4}K$ _____ 9. $2\frac{2}{3}K$ _____

10. Reasonableness What is a reasonable estimate for $7\frac{3}{4} \times 2\frac{2}{3}$?
 Explain.

11. The cups of mushrooms in a recipe is $2\frac{1}{2}$ times the cups of onions.
 The cups of onions is $1\frac{1}{2}$. Solve $c = 1\frac{1}{2} \times 2\frac{1}{2}$ to find c , the cups of mushrooms.

Understanding Division of Fractions

Divide a fraction by a whole number.

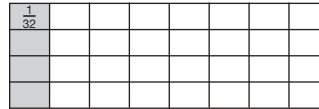
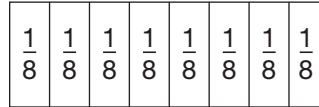
Find $\frac{1}{8} \div 4$.

Use a model to show $\frac{1}{8}$.

Divide each eighth into 4 equal parts.

Each section shows $\frac{1}{(8 \times 4)} = \frac{1}{32}$.

$$\frac{1}{8} \div 4 = \frac{1}{32}$$

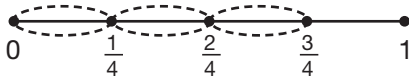


Divide a fraction by a fraction.

Find $\frac{3}{4} \div \frac{1}{4}$.

Use a number line.

Count the number of $\frac{1}{4}$ s in $\frac{3}{4}$.



There are three $\frac{1}{4}$ s.

$$\frac{3}{4} \div \frac{1}{4} = 3$$

Use repeated subtraction.

Subtract $\frac{1}{4}$ from $\frac{3}{4}$ until the difference is 0.

$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

$$\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$$

$$\frac{1}{4} - \frac{1}{4} = 0$$

Count the number of times you subtracted to find the quotient.

$$\frac{3}{4} \div \frac{1}{4} = 3$$

Solve each division sentence. Use a model if you wish.

1. $3 \div \frac{1}{3} =$ _____

2. $\frac{1}{5} \div 4 =$ _____

Find each quotient. Simplify if possible.

3. $3 \div \frac{1}{2} =$ _____

4. $\frac{9}{10} \div \frac{1}{10} =$ _____

5. $\frac{1}{5} \div 3 =$ _____

6. $\frac{3}{16} \div \frac{1}{16} =$ _____

7. $5 \div \frac{1}{3} =$ _____

8. $\frac{1}{2} \div 6 =$ _____

9. $8 \div \frac{1}{4} =$ _____

10. $\frac{7}{12} \div \frac{1}{12} =$ _____

11. $\frac{6}{7} \div \frac{1}{7} =$ _____

12. **Draw a Picture** The square dancing club meets for 3 hours. Every $\frac{3}{4}$ hour, the dancers change partners. How many different partners will each dancer have in one meeting? Draw a picture to show your solution.

13. **Writing to Explain** Explain why the quotient of two fractions less than 1 is always greater than either fraction.

Dividing Fractions

To divide by a fraction, you can multiply by its reciprocal. The reciprocal of a number has the numerator and the denominator reversed.

Find $\frac{4}{5} \div \frac{3}{10}$.

Step 1

Rewrite the division as multiplication using the reciprocal of the divisor.

The reciprocal of $\frac{3}{10}$ is $\frac{10}{3}$.

$$\frac{4}{5} \div \frac{3}{10} = \frac{4}{5} \times \frac{10}{3}$$

Step 2

Divide out common factors if possible. Then multiply.

$$\frac{\cancel{4}^2}{5} \times \frac{\cancel{10}^2}{3} = \frac{8}{3}$$

Step 3

If your answer is an improper fraction, change it to a mixed number.

$$\frac{8}{3} = 2\frac{2}{3}$$

Find each quotient. Simplify if possible.

1. $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{\uparrow}{\text{Reciprocal of } \frac{1}{4}} = \underline{\hspace{2cm}}$

2. $\frac{4}{7} \div \frac{8}{21} = \underline{\hspace{2cm}} \times \frac{\uparrow}{\text{Reciprocal of } \frac{8}{21}} = \underline{\hspace{2cm}}$

3. $\frac{1}{3} \div \frac{1}{2} = \underline{\hspace{2cm}}$

4. $\frac{2}{5} \div \frac{2}{3} = \underline{\hspace{2cm}}$

5. $\frac{5}{8} \div \frac{7}{10} = \underline{\hspace{2cm}}$

6. $\frac{3}{7} \div 3 = \underline{\hspace{2cm}}$

7. $\frac{1}{3} \div \frac{8}{9} = \underline{\hspace{2cm}}$

8. $\frac{5}{6} \div \frac{1}{8} = \underline{\hspace{2cm}}$

9. $\frac{5}{9} \div \frac{1}{2} = \underline{\hspace{2cm}}$

10. $\frac{3}{5} \div \frac{3}{4} = \underline{\hspace{2cm}}$

11. $\frac{3}{4} \div \frac{5}{6} = \underline{\hspace{2cm}}$

12. $\frac{9}{10} \div \frac{4}{5} = \underline{\hspace{2cm}}$

13. $\frac{1}{3} \div \frac{3}{8} = \underline{\hspace{2cm}}$

14. $\frac{4}{7} \div \frac{3}{4} = \underline{\hspace{2cm}}$

15. Aaron has $\frac{7}{8}$ gallon of bottled water. How many $\frac{3}{16}$ -gallon servings can he pour?
- _____

16. **Draw a Picture** Show how Rebecca can divide $\frac{3}{4}$ of a cake into 9 pieces. What fraction of the whole cake will each piece be?
- _____

Dividing Mixed Numbers

You can follow these steps to find $5\frac{1}{3} \div 1\frac{1}{3}$ and $21 \div 2\frac{1}{3}$.

Step 1	Step 2	Step 3
<p>First estimate. Then write each number as an improper fraction.</p> <p>Find $5\frac{1}{3} \div 1\frac{1}{3}$. Estimate $5 \div 1 = 5$.</p> $5\frac{1}{3} \div 1\frac{1}{3} =$ $\begin{array}{r} \downarrow \quad \downarrow \\ \frac{16}{3} \div \frac{4}{3} \end{array}$	<p>Find the reciprocal of the divisor. Rewrite as a multiplication problem.</p> $\frac{16}{3} \div \frac{4}{3} =$ $\frac{16}{3} \times \frac{3}{4}$	<p>Look for common factors. Simplify, then multiply.</p> $\frac{16}{3} \times \frac{3}{4} =$ $\frac{\overset{4}{\cancel{16}}}{1} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{4}} = \frac{4}{1} = 4$ <p>4 is close to 5, so the answer is reasonable.</p>
<p>Find $21 \div 2\frac{1}{3}$. Estimate $21 \div 2 = 10\frac{1}{2}$.</p> $21 \div 2\frac{1}{3}$ $\begin{array}{r} \downarrow \quad \downarrow \\ \frac{21}{1} \div \frac{7}{3} \end{array}$	$\frac{21}{1} \div \frac{7}{3} =$ $\frac{21}{1} \times \frac{3}{7}$	$\frac{21}{1} \times \frac{3}{7} =$ $\frac{\overset{3}{\cancel{21}}}{1} \times \frac{\underset{1}{\cancel{3}}}{7} = \frac{9}{1} = 9$ <p>9 is close to $10\frac{1}{2}$, so the answer is reasonable.</p>

Find each quotient. Simplify if possible.

1. $2\frac{2}{3} \div 3\frac{1}{4} =$ _____

2. $1\frac{3}{4} \div 4\frac{1}{8} =$ _____

3. $2\frac{1}{5} \div 2\frac{1}{3} =$ _____

4. $5\frac{1}{4} \div 3 =$ _____

5. $10 \div 3\frac{1}{4} =$ _____

6. $7\frac{1}{4} \div 2\frac{1}{8} =$ _____

7. **Writing to Explain** Paper needs to be cut for voting ballots.

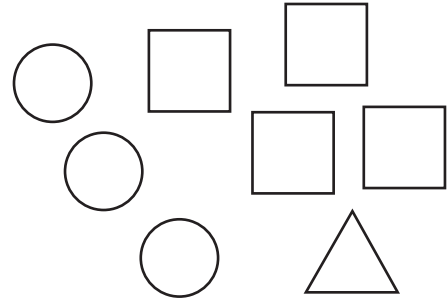
Each piece of paper is $10\frac{1}{2}$ in. long. Each ballot should be $1\frac{3}{4}$ in. long. How many ballots can be cut from one piece of paper?

Understanding Ratios

A ratio is a pair of numbers that compares two quantities.

Count to find the ratio of squares to circles.

↓ ↓
 4 to 3



The ratio 4 to 3 can also be written as 4:3 or $\frac{4}{3}$.

The order of the numbers in a ratio is important.

4:3 is the ratio of squares to circles.

3:4 is the ratio of circles to squares.

Use the picture above for exercises 1 through 6. Write a ratio for each comparison in three ways.

1. The number of triangles to the total number of shapes

↓ ↓
 1 to 8

2. The number of squares to the number of triangles

3. The number of triangles to the number of squares

4. The number of triangles to the number of circles

5. The number of circles to the total number of shapes

6. The total number of shapes to the number of squares

7. There are 14 boys and 16 girls in Mr. Allen's class. What is the ratio of girls to the total number of students in the class? Write the ratio 3 ways.

8. **Writing to Explain** At a cat and dog hospital, 9 of the patients were cats, 17 were dogs. Use this fact to write two ratios. Explain what each ratio means.

Equal Ratios and Proportions

You can find equal ratios just like you find equivalent fractions.

Find ratios equal to $\frac{30}{40}$.

Multiply both terms by the same number.

$$\frac{30 \times 2}{40 \times 2} = \frac{60}{80}$$

Divide both terms by the same number. To find the simplest form ratio, divide by the greatest common factor (GCF) of the two numbers.

The GCF of 30 and 40 is 10.

$$\frac{30 \div 10}{40 \div 10} = \frac{3}{4}$$

Two equal ratios form a proportion. The units must be the same in both ratios.

Do the ratios 24 ft:16 seconds and 36 ft:24 seconds form a proportion?

First check the units.

Both ratios compare feet to seconds, so the units are the same.

Then write each ratio in simplest form.

$$\frac{24 \text{ ft}}{16 \text{ seconds}} = \frac{3 \text{ ft}}{2 \text{ seconds}}$$

$$\frac{36 \text{ ft}}{24 \text{ seconds}} = \frac{3 \text{ ft}}{2 \text{ seconds}}$$

Compare the simplest form ratios.

They are the same, so the ratios form a proportion.

Write three ratios that are equal to the ratio given.

1. $\frac{3}{5}$ _____ 2. $\frac{4}{8}$ _____ 3. $\frac{6}{18}$ _____

4. 8:10 _____ 5. 6:8 _____ 6. 10:12 _____

7. 12 to 18 _____ 8. 16 to 18 _____ 9. 5 to 25 _____

Write the ratios in simplest form.

10. $\frac{10}{15}$ _____ 11. 21 to 14 _____ 12. 15:25 _____

Write = if the ratios form a proportion; if they do not form a proportion, write \neq .

13. $\frac{15}{18} \mid \frac{10}{12}$ _____ 14. 20:24 \mid 24:30 _____ 15. 16 to 20 \mid 28 to 35 _____

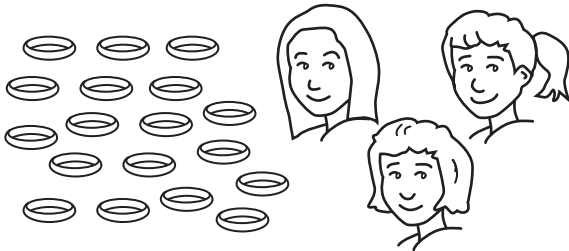
16. **Number Sense** Dale says that the ratios 3:5 and 2:10 are equal. Is he correct? Explain.

Understanding Rates and Unit Rates

A rate is a ratio in which the two terms are measured in different units.

Example: 18 bracelets for 3 girls.

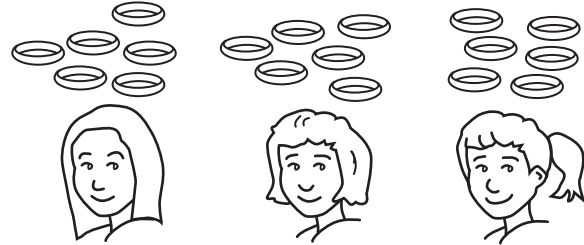
$$\frac{18 \text{ bracelets}}{3 \text{ girls}}$$



In a unit rate, the second number is 1.

Example: 6 bracelets for 1 girl.

$$\frac{6 \text{ bracelets}}{1 \text{ girl}}$$



Remember that the fraction bar shows division.
If you know a rate, you can divide to find the unit rate.

Example: 17 goals in 5 games is written as $\frac{17 \text{ goals}}{5 \text{ games}}$.

$$\begin{array}{r} 3.4 \\ 5 \overline{)17.0} \end{array}$$

The unit rate is 3.4 goals per game. (Per means "for each".)

Write the rate and the unit rate.

1. 25 flowers for 5 vases

2. 32 games in 8 weeks

3. 144 pencils in 12 packages

4. 252 students in 9 classes

5. \$13.20 for 6 pounds

6. 34 minutes for 8 pages

7. **Number Sense** If a car travels 350 miles in 7 hours, what is its rate per hour?

8. **Estimation** Bare root plum trees are on sale at 3 for \$40. To the nearest dollar, what is the cost per tree?

Using Unit Rates

A unit rate is a special ratio that compares one quantity to one unit of another quantity. You can use unit rates to solve proportions.

Geraldo earns \$100 for 4 hours of work. If he works 7 hours at the same rate of pay, how much will he earn?

Write a proportion.

Use d for dollars earned.

$$\frac{\$100}{4 \text{ h}} = \frac{\$d}{7 \text{ h}}$$

Find the unit rate.

Divide the first term by the second term.

$$\$100 \div 4 = \$25$$

The unit rate is $\frac{\$25}{1 \text{ h}}$.

Think: Find an equal ratio with 1 as the second term.

$$\frac{100 \div 4}{4 \div 4} = \frac{25}{1}$$

Multiply by the unit rate.

$$\frac{\$25}{\text{h}} \times 7 \text{ h} = \$175$$

Think: Find an equal ratio.

$$\frac{25 \times 7}{1 \times 7} = \frac{175}{7}$$

So, $\frac{\$100}{4 \text{ h}} = \frac{\$175}{7 \text{ h}}$. Geraldo will earn \$175 when he works 7 hours.

Use unit rates to solve each proportion. Estimate to check reasonableness.

1. $\frac{\boxed{} \text{ g}}{2 \text{ kg}} = \frac{30 \text{ g}}{15 \text{ kg}}$

Unit Rate: _____

Multiply: _____

2. $\frac{120 \text{ mi}}{3 \text{ gal}} = \frac{\boxed{} \text{ mi}}{5 \text{ gal}}$

Unit Rate: _____

Multiply: _____

3. $\frac{8 \text{ in.}}{2 \text{ wk}} = \frac{\boxed{} \text{ in.}}{5 \text{ wk}}$

Unit Rate: _____

Multiply: _____

4. $\frac{\$24}{3 \text{ wk}} = \frac{\$ \boxed{}}{10 \text{ wk}}$

5. $\frac{\boxed{} \text{ oz}}{7 \text{ packs}} = \frac{64 \text{ oz}}{8 \text{ packs}}$

6. $\frac{200 \text{ stamps}}{2 \text{ rows}} = \frac{\boxed{} \text{ stamps}}{9 \text{ rows}}$

7. Wes used 49 quarts of oil when he changed the oil in 7 cars. Complete and solve the proportion to find how many quarts of oil he would use to change the oil in 20 cars, assuming that all cars need the same quantity of oil.

$$\frac{49 \text{ quarts}}{7 \text{ cars}} = \frac{}{}$$

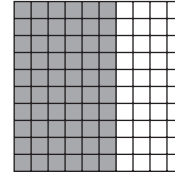
8. **Writing to Explain** A café served 180 pickles with 60 sandwiches. If the ratio of sandwiches to pickles is always constant, explain how you can use unit rates and proportions to find how many pickles are needed to serve 32 sandwiches.

Understanding Percent

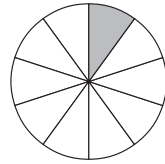
A percent is a ratio that compares a part to a whole.
The second term in the ratio is always 100.
The whole is 100%.
The grid has 60 of 100 squares shaded.

$$\frac{60}{100} = 60\%$$

So, 60% of the grid is shaded.



When the second term of a ratio is not 100, you can write an equivalent ratio with a denominator of 100 or use a proportion to find the percent shown by the part.



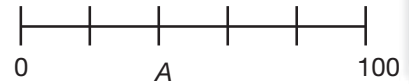
$$\frac{1}{10} = \frac{10}{100} = 10\% \quad \text{or} \quad \frac{1}{10} = \frac{x}{100}$$

$$10x = 100$$

$$x = 10$$

So, 10% of the circle is shaded.

The line segment represents 100%.
What percent is shown by Point A?



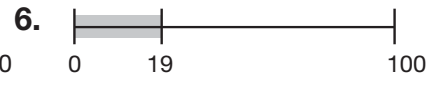
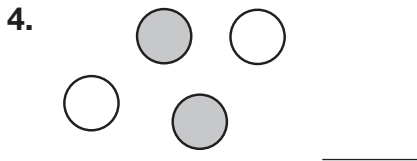
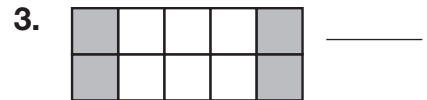
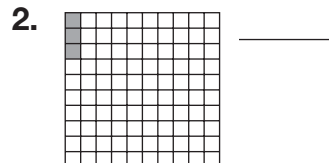
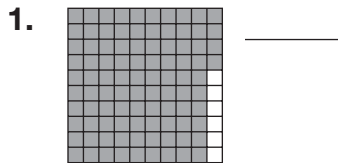
$$\frac{2}{5} = \frac{40}{100} = 40\% \quad \text{or} \quad \frac{2}{5} = \frac{x}{100}$$

$$5x = 200$$

$$x = 40$$

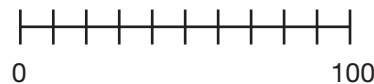
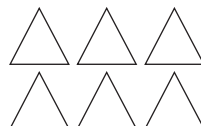
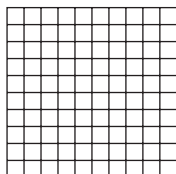
So, 40% of the line segment is shaded.

Write the percent of each figure that is shaded.



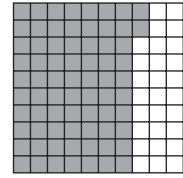
7. **Number Sense** Jana divided a sheet of paper into 5 equal sections and colored 2 of the sections red. What percent of the paper did she color? _____

8. **Writing to Explain** Shade each model to show 100%. Explain how you knew how many parts to shade.



Fractions, Decimals, and Percents

Fractions, decimals, and percents all name parts of a whole. Percent means per hundred, so 15% means 15 parts per hundred. The grid to the right has 72 out of 100 squares shaded. The shaded part can be represented with a fraction, $\frac{72}{100}$ ($\frac{18}{25}$ in simplest form), by a decimal, 0.72, and by a percent, 72%.



Write 36% as a fraction in simplest form and as a decimal.

$$36\% = \frac{36}{100} = 0.36$$

Simplify the fraction:

$$\frac{36}{100} = \frac{36 \div 4}{100 \div 4} = \frac{9}{25}$$

$$\text{So, } 36\% = \frac{9}{25} = 0.36.$$

Write 0.47 as a fraction in simplest form and as a percent.

$$0.47 = \frac{47}{100} = 47\%$$

Write $\frac{3}{4}$ as a decimal and as a percent.

You can use a proportion or divide to help you.

Use a proportion:

$$\begin{aligned} \frac{3}{4} &= \frac{n}{100} \\ 4n &= 300 \\ n &= 75 \end{aligned}$$

Use division:

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\text{So, } \frac{3}{4} = \frac{75}{100} = 0.75 = 75\%.$$

Write each number in two other ways. Write fractions in simplest form.

1. $\frac{2}{100}$ _____ ; _____

2. $\frac{71}{100}$ _____ ; _____

3. $\frac{9}{10}$ _____ ; _____

4. 17% _____ ; _____

5. 48% _____ ; _____

6. 60% _____ ; _____

7. 0.04 _____ ; _____

8. 0.22 _____ ; _____

9. **Writing to Explain** Jamal said that he could write a percent as a decimal by moving the decimal point two places to the left and deleting the percent sign. Is he correct? How do you know?

10. **Number Sense** Two stores sell their goods at the manufacturers' suggested retail prices, so their prices are the same. Which store has the greatest markdown from their original prices?

GOODS 2 GO
 $\frac{1}{4}$ off
original prices!

BUY AND BYE
30% off
original prices!

Finding the Percent of a Number

Find 77% of 240.

First estimate.

$$77\% \approx 75\% = \frac{3}{4}$$

$$\frac{3}{4} \times 240 = 180$$

Use a decimal.

Change the percent to a decimal.

$$77\% = 0.77$$

Multiply.

$$0.77 \times 240 = 184.8$$

The answer 184.8 is close to the estimate 180.

Use a proportion.

Write the percent as a fraction.

$$77\% = \frac{77}{100}$$

Write the proportion and solve.

$$\frac{x}{240} = \frac{77}{100}$$

$$100x = 18,480$$

$$\frac{100x}{100} = \frac{18,480}{100}$$

$$x = 184.8$$

Find the percent of each number.

- | | | |
|---------------------|----------------------|----------------------|
| 1. 25% of 24 _____ | 2. 50% of 72 _____ | 3. 72% of 88 _____ |
| 4. 18% of 97 _____ | 5. 66% of 843 _____ | 6. 46% of 388 _____ |
| 7. 89% of 111 _____ | 8. 0.7% of 392 _____ | 9. 110% of 640 _____ |

- 10. Geometry** Ava's aquarium is 10 in. tall, 15 in. long, and 8 in. wide. The aquarium is 95% filled with water. How many cubic inches of water are in the aquarium?
- _____

- 11.** DeWayne used his music club membership card to get 15% off the cost of a CD. If the regular price of the CD was \$15.95, how much did DeWayne pay?
- _____

- 12.** Marla bought a dress priced at \$89.99. She used a 20% off coupon. How much did she pay for the dress?
- _____

- 13. Writing to Explain** Tell how you could use a proportion to find 125% of 500. Why is the solution greater than the original number?
- _____

Equations with More Than One Operation

Some equations require more than one operation to solve. When you solve an equation with more than one step, undo the operations in this order:

- First undo addition or subtraction.
- Then undo multiplication or division.

<p>Solve $5x - 10 = 95$.</p> <p>Step 1: Undo subtraction. Add 10 to both sides.</p> <p>Step 2: Undo multiplication. Divide both sides by 5.</p> <p>Step 3: Check by substitution.</p>	$5x - 10 = 95$ $5x - 10 + 10 = 95 + 10$ $5x = 105$ $\frac{5x}{5} = \frac{105}{5}$ $x = 21$ $5x - 10 = 95$ $5(21) - 10 = 95$ $105 - 10 = 95$ $95 = 95 \checkmark$
<p>Solve $10 = \frac{n}{5} + 6$</p> <p>Step 1: Undo addition. Subtract 6 from both sides.</p> <p>Step 2: Undo division. Multiply both sides by 5.</p> <p>Step 3: Check by substitution.</p>	$10 = \frac{n}{5} + 6$ $10 - 6 = \frac{n}{5} + 6 - 6$ $4 = \frac{n}{5}$ $4 \times 5 = \frac{5 \times n}{5}$ $20 = n$ $10 = \frac{n}{5} + 6$ $10 = \frac{20}{5} + 6$ $10 = 4 + 6$ $10 = 10 \checkmark$

Solve each equation and check your solution.

1. $8b + 16 = 64$ _____

2. $2y - 4 = 24$ _____

3. $\frac{q}{10} + 5 = 10$ _____

4. $\frac{m}{3} + 2 = 17$ _____

5. $\frac{p}{4} + 13 = 21$ _____

6. $5b - 8 = 17$ _____

7. $\frac{a}{3} - 17 = 14$ _____

8. $3d + 17 = 24.5$ _____

9. **Number Sense** Would you expect the solution of $4x + 12 = 36$ to be greater than or less than 36? Explain.
- _____